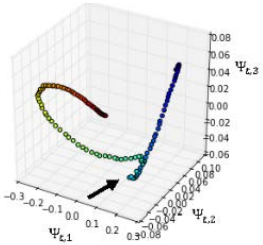
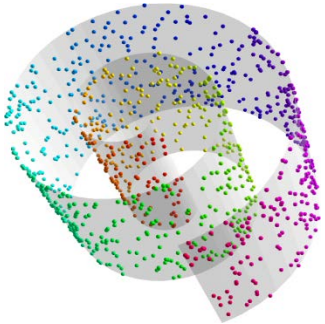
Geometry Learning for Neuronal Data Analysis: Hierarchical Coupled Geometry

Ron Meir
Electrical Engineering
Technion



Data Analysis in High Dimensions

- When can complex data be simplified?
  - Smoothness
  - Sparsity
  - Geometry
- Suggestion: Data lies on low-dim manifold
  - Learn manifold from data
  - Advantage: work in low-dim space
  - Many approaches: Laplacian eigenmaps, Hessian eigenmaps, diffusion maps (Coifman & Lafon 06), ...



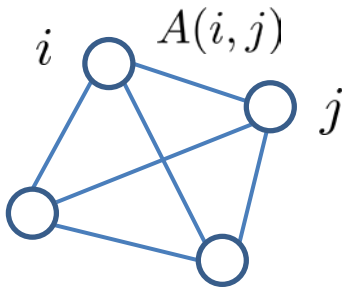
Manifold Learning

(Diffusion map)

Data $X = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M\}$ $\boldsymbol{x}_i \in \mathbb{R}^N$

- Define affinity matrix $A(i,j) = \exp\left(-\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2/\epsilon\right)$

- Construct weighted graph from data



- Normalize affinity matrix $P(i,j) = A(i,j)/\sum_j A(i,j)$

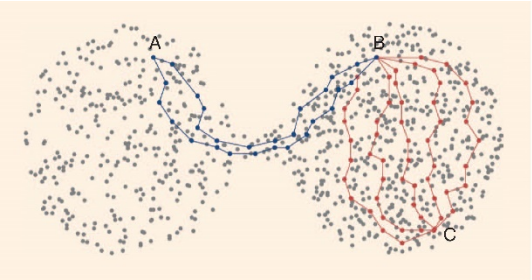
- Eigenvalue decomposition

$$1 = \lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_M > 0$$

$$\mathbf{1} \quad \psi_1 \quad \cdots \quad \psi_M$$

Low Dimensional Representation

$D(i,j)$ Global metric



Coifman and Lafon 2006

Question How can it be constructed?

Mapping $\mathbb{R}^N \mapsto \mathbb{R}^n$ Nonlinear

$$\boldsymbol{x}_i \mapsto (\lambda_1\psi_1(i), \ldots, \lambda_n\psi_n(i)) \triangleq \Psi(i)$$

Low-dim representation $n \ll N$

Claim

$$D(i,j) = \|\Psi(i) - \Psi(j)\|$$

$$n = M - 1$$

Benefits

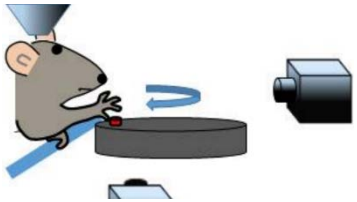
- Global metric using local means
- Euclidean space
- Often low-dim $n \ll N$

- Excellent empirical results
- Strong theory
- Noise robustness

Experiment Description

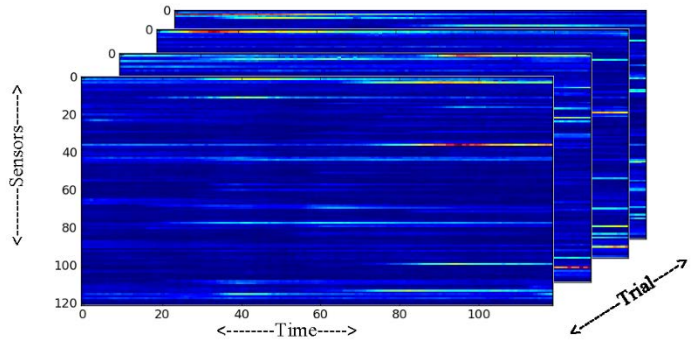
- **Neuronal activity recordings** [Schiller's Lab, Technion]
  - Two-photon in-vivo calcium imaging
  - High resolution in time and space
  - Identified neurons over the course of days and weeks

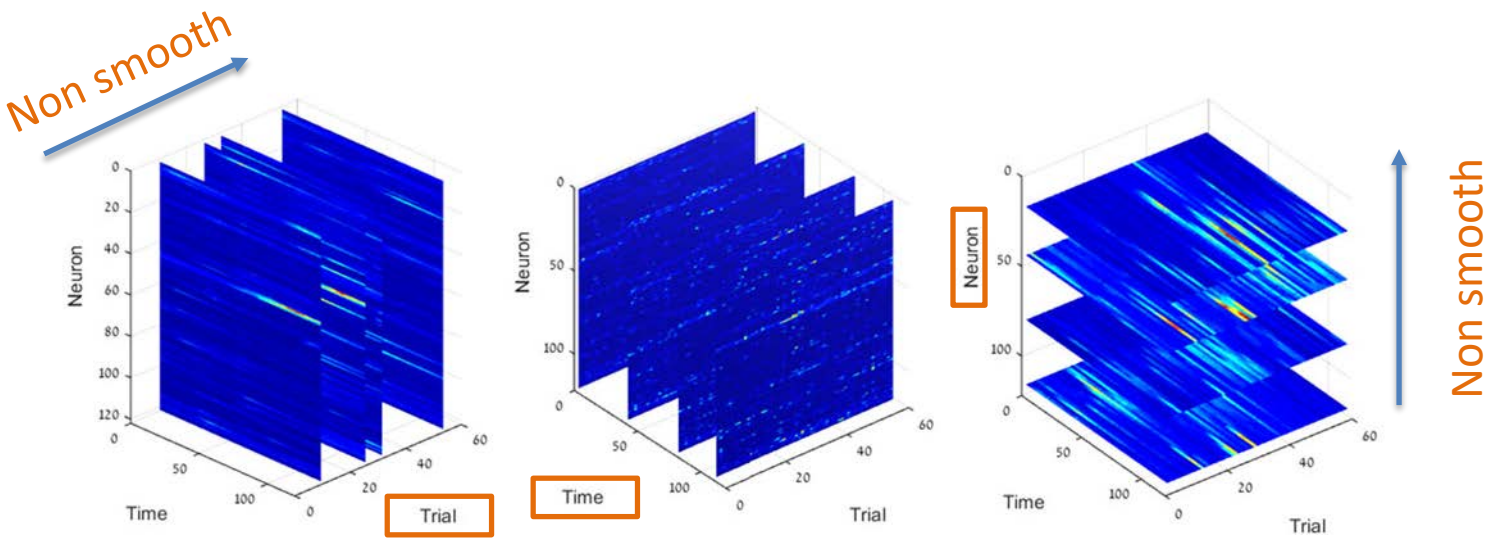
- **Protocol**
  - Repeated trials motor forepaw movement
  - **Task:** reach pellet upon tone, in mouth
  - Behavior monitoring
  - Silencing of somato-sensory cortex possible



Experimental Data

X





X_T X_t X_r

Hierarchical Coupled Geometry Analysis I

Objective

Construct hierarchical multiscale metric for data

Problem

What is an appropriate distance measure?

Algorithm

- Select an initial (local) metric for each dimension
- Iteratively refine metric using all data - coupled
  Output is a tree for each dimension
  - Based on a metric coupling all dimensions

Mishne et al., Hierarchical Coupled-Geometry Analysis for Neuronal Structure and Activity Pattern Discovery, 2016

Hierarchical Coupled Geometry Analysis II

Initialization

Input 3D data matrix X

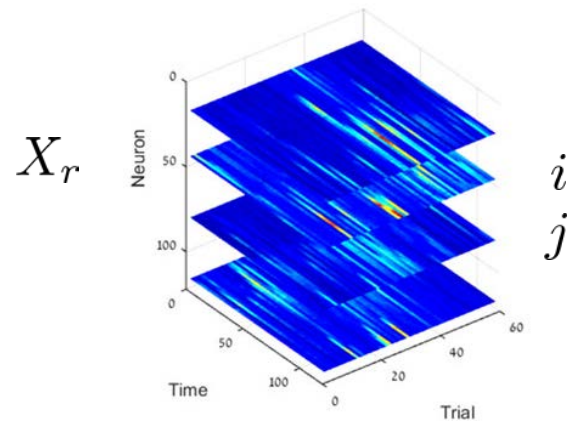
Starting with neuron dimension r

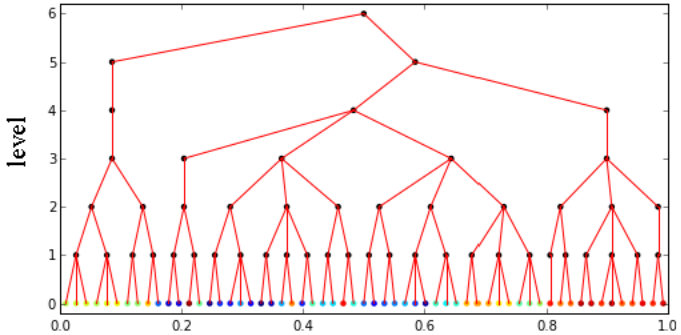
- Calculate initial affinity $A_r^{(0)}(i,j)$
- Calculate initial embedding $\Psi_r^{(0)}$
- Calculate initial tree $\mathcal{T}_r^{(0)}$

Repeat for time dimension, $\mathcal{T}_t^{(0)}$

$\mathcal{T}_r^{(0)}, \mathcal{T}_t^{(0)}$

Initial neuron and time trees





Hierarchical Coupled Geometry Analysis III

Neuron tree + Time tree → Inter trial distance

Iterative 3D analysis

Input $\mathcal{T}_r^{(0)}, \mathcal{T}_t^{(0)}$

for $n \geq 1$ **do**

Calculate multi-scale bi-tree distance between trials $d^{(n)}(T_i, T_j)$

Calculate trial affinity $A_T^{(n)}(i,j) = \exp\left(-d^{(n)}(T_i, T_j)/\sigma_T\right)$

Calculate trial embedding $\Psi_T^{(n)}$

Calculate trial tree $\mathcal{T}_T^{(n)}$

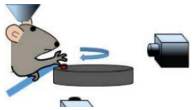
Similarly for neuron and time trees

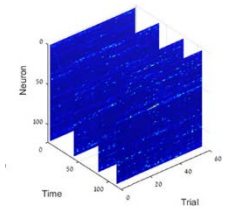
end for

$\mathcal{T}_r^*,\ \mathcal{T}_t^*,\ \mathcal{T}_T^*$

Hierarchical multiscale flexible trees

Embedding of Time Frames





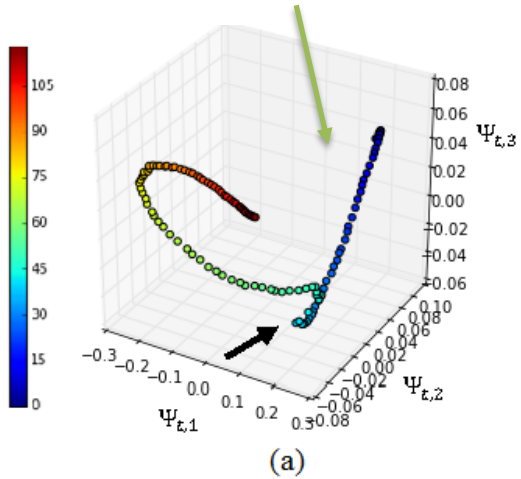
Time evolution

First 11 eigenvectors

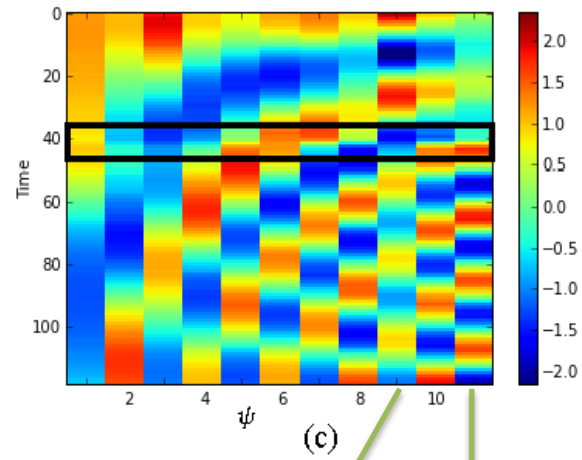
Data-dependent harmonic functions

Before tone

Time index



Tone



Tone (t=42) – discovered from data

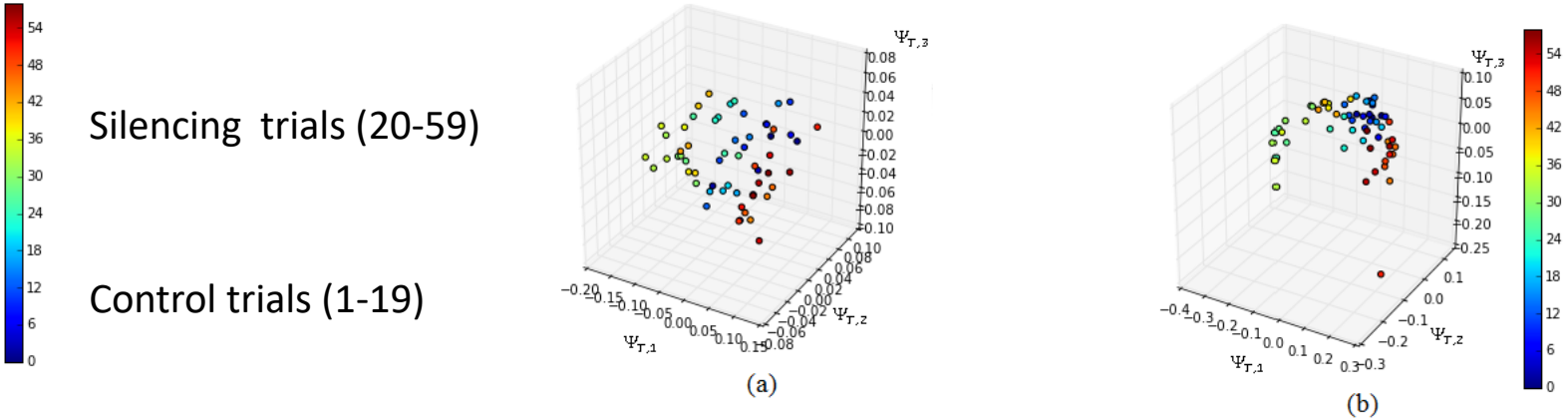
Localized pre-tone

Localized post-tone

Embedding of Trial Slices I

Silencing trials (20-59)

Control trials (1-19)

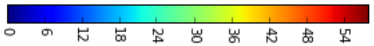


Initial

Iterated

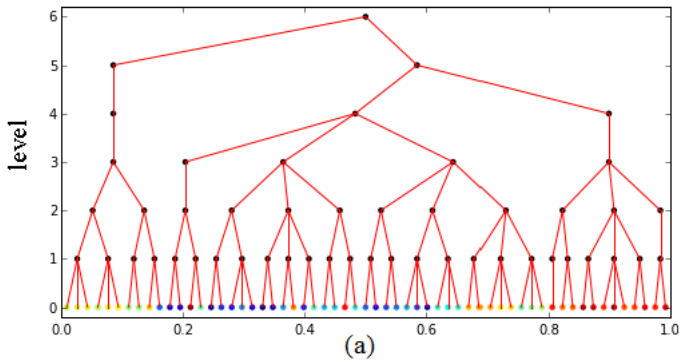
- Trial data clustered into 3 groups (1-19, 20-44, 45-59)
- Detect pathological dysfunction
- Effect expires during experiment

Embedding of Trial Slices II

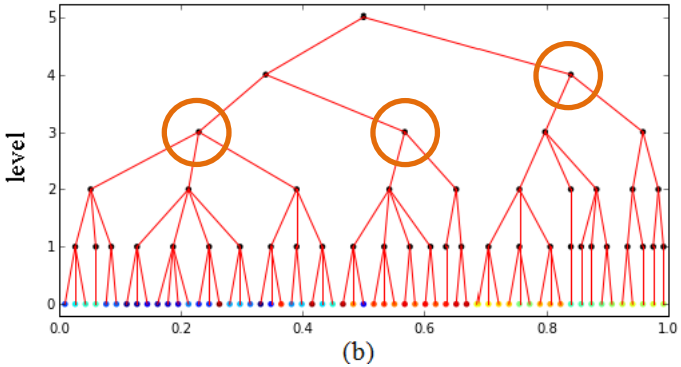


Trials

Trials



Initial



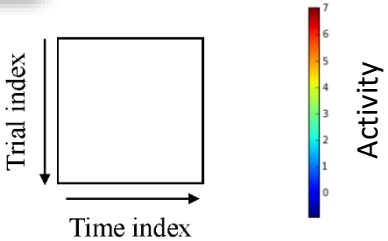
Iterated

Captures structure of trials

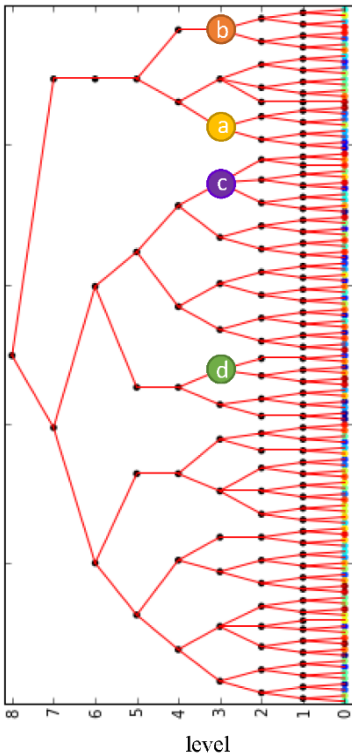
Neuronal Analysis

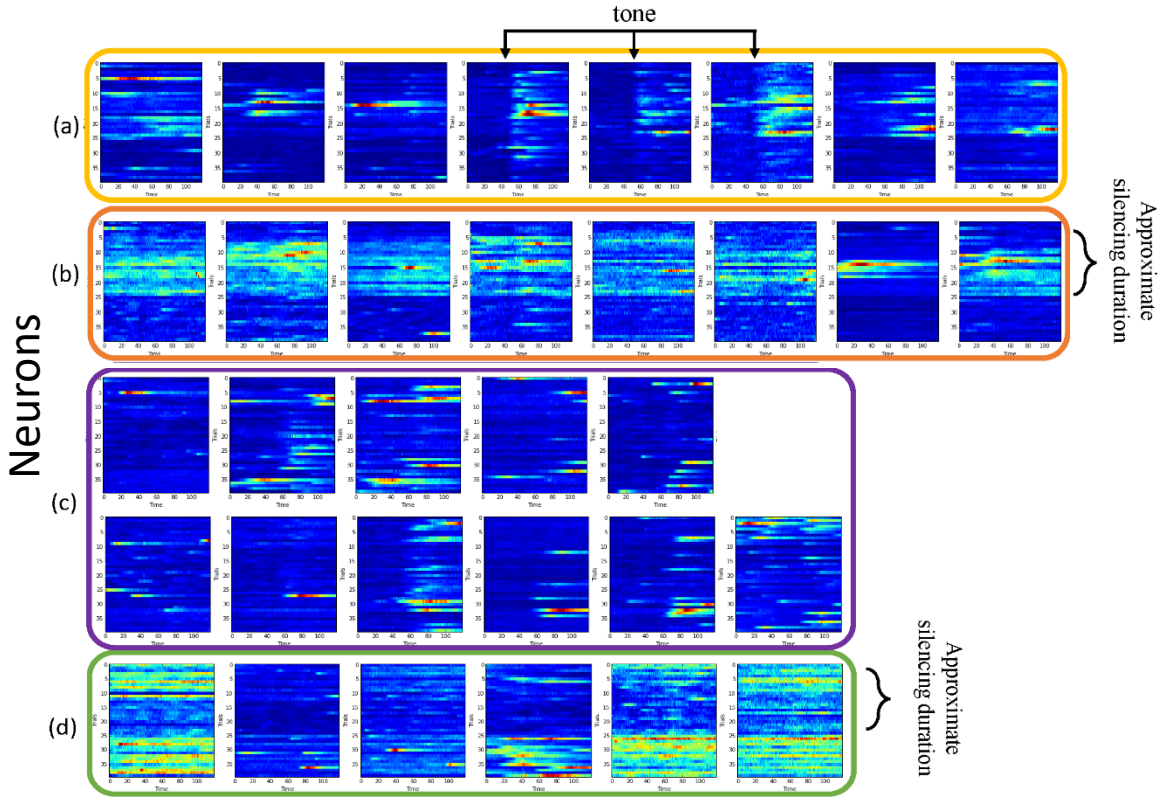
Silencing trials $20-44,\ 45-59$

S1 S2



Neuron tree





After tone

Under S1

Under S1

After tone

Under S2

Silenced

Summary

- New data-dependent global analysis of trial data
- Iterative local to global refinement procedure
  - Coupled hierarchical organization
  - Global embedding in each dimension
- Identified meaningful patterns from data alone
  - Related to external and internal triggers
- Constructed data-dependent dictionaries

Future

- More dimensions – e.g., add behavior axis
- Interaction between sensory and motor components
- Use results as basis for data-motivated models

Acknowledgements

- Ronen Talmon
- Raphy Coifman
- Jackie Schiller
- Gal Mishne
- Hadas Benisty