Geometry Learning for Neuronal Data Analysis: Hierarchical Coupled Geometry

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Data Analysis in High Dimensions

- When can complex data be simplified?
  - Smoothness
  - Sparsity
  - Geometry

**Suggestion:** Data lies on low-dim manifold
  - Learn manifold from data
  - Advantage: work in low-dim space
  - Many approaches: Laplacian eigenmaps, Hessian eigenmaps, **diffusion maps** (Coifman & Lafon 06), ...
Manifold Learning

Data \( X = \{ \mathbf{x}_1, \ldots, \mathbf{x}_M \} \quad \mathbf{x}_i \in \mathbb{R}^N \)

- Define affinity matrix

\[ A(i, j) = \exp \left( -\| \mathbf{x}_i - \mathbf{x}_j \|^2 / \epsilon \right) \]

- Construct weighted graph from data

- Normalize affinity matrix

\[ P(i, j) = A(i, j) / \sum_j A(i, j) \]

- Eigenvalue decomposition

\[ 1 = \lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_M > 0 \]

\[ \begin{bmatrix} 1 & \psi_1 & \cdots & \psi_M \end{bmatrix} \]
Low Dimensional Representation

$D(i, j)$ Global metric

Question

How can it be constructed?

Mapping

$\mathbb{R}^N \rightarrow \mathbb{R}^n$ Nonlinear

$\mathbf{x}_i \mapsto (\lambda_1 \psi_1(i), \ldots, \lambda_n \psi_n(i)) \triangleq \Psi(i)$

Low-dim representation $n \ll N$

Claim

$D(i, j) = \|\Psi(i) - \Psi(j)\|$

$n = M - 1$

Benefits

Global metric using local means

Euclidean space

Often low-dim $n \ll N$

Excellent empirical results

Strong theory

Noise robustness
Experiment Description

**Neuronal activity recordings** [Schiller’s Lab, Technion]
- Two-photon in-vivo calcium imaging
- High resolution in time and space
- Identified neurons over the course of days and weeks

**Protocol**
- Repeated trials motor forepaw movement
- **Task:** reach pellet upon tone, in mouth
- Behavior monitoring
- **Silencing** of somato-sensory cortex possible
Experimental Data

$X$

$X_T$

$X_t$

$X_r$

Non smooth
Hierarchical Coupled Geometry Analysis I

Objective

Construct hierarchical multiscale metric for data

Problem

What is an appropriate distance measure?

Algorithm

- Select an initial (local) metric for each dimension
- Iteratively refine metric using all data - coupled

Output is a tree for each dimension
- Based on a metric coupling all dimensions

Mishne et al., Hierarchical Coupled-Geometry Analysis for Neuronal Structure and Activity Pattern Discovery, 2016
Initialization

**Input** 3D data matrix $X$

Starting with neuron dimension $r$

- Calculate initial affinity $A_r^{(0)}(i, j)$
- Calculate initial embedding $\Psi_r^{(0)}$
- Calculate initial tree $T_r^{(0)}$

Repeat for time dimension, $T_t^{(0)}$

$T_r^{(0)}, T_t^{(0)}$

Initial neuron and time trees
Hierarchical Coupled Geometry Analysis III

Iterative 3D analysis

Input $\mathcal{T}_r^{(0)}, \mathcal{T}_t^{(0)}$

for $n \geq 1$ do

  Calculate multi-scale bi-tree distance between trials $d^{(n)}(T_i, T_j)$
  Calculate trial affinity $A_T^{(n)}(i, j) = \exp\left(-d^{(n)}(T_i, T_j)/\sigma_T\right)$
  Calculate trial embedding $\Psi_T^{(n)}$
  Calculate trial tree $\mathcal{T}_T^{(n)}$

Similarly for neuron and time trees

end for

$\mathcal{T}_r^*, \mathcal{T}_t^*, \mathcal{T}_T^*$

Hierarchical multiscale flexible trees
Embedding of Time Frames

Time evolution

First 11 eigenvectors

Data-dependent harmonic functions

Before tone

Tone

Tone (t=42) – discovered from data

Localized pre-tone

Localized post-tone
Embedding of Trial Slices I

Silencing trials (20-59)

Control trials (1-19)

- Trial data clustered into 3 groups (1-19, 20-44, 45-59)
- Detect pathological dysfunction
- Effect expires during experiment
Embedding of Trial Slices II

(a) Initial

(b) Iterated

Captures structure of trials
Neuronal Analysis

**Silencing trials**  20 – 44, 45 – 59

S1  S2

Neuron tree

Neurons

After tone
Under S1

After tone
Under S2

Silenced
Summary

- New data-dependent global analysis of trial data
- Iterative local to global refinement procedure
  - Coupled hierarchical organization
  - Global embedding in each dimension
- Identified meaningful patterns from data alone
  - Related to external and internal triggers
- Constructed data-dependent dictionaries

Future

- More dimensions – e.g., add behavior axis
- Interaction between sensory and motor components
- Use results as basis for data-motivated models
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