Decoding the population activity of grid cells for spatial localization & goal-directed navigation

CRCNS Meeting – Paris, October 2016

Andreas Herz

Alexander Mathis

Martin Stemmler

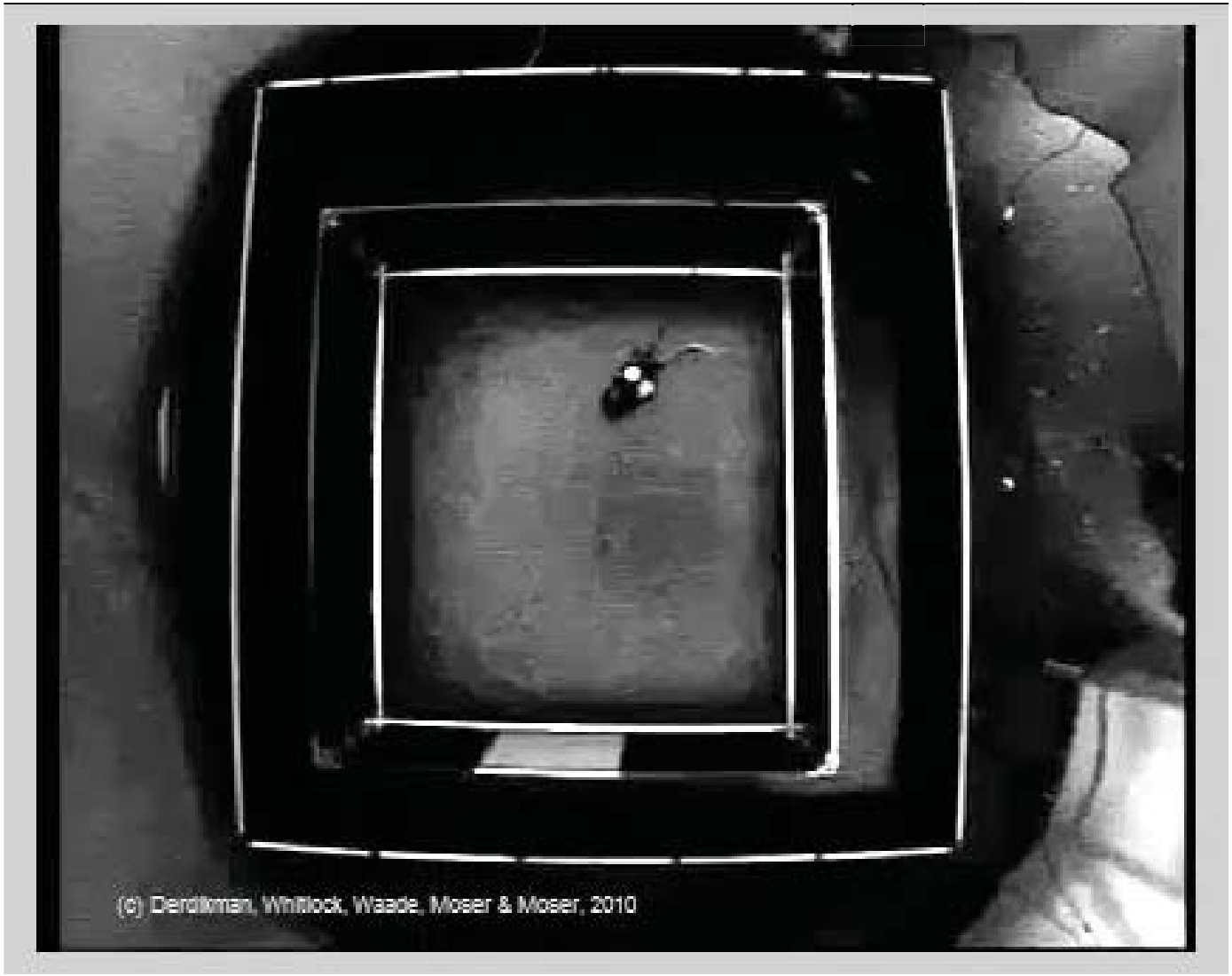
Stemmler / Mathis / Herz
Science Advances 2015

Ludwig-Maximilians-Universität

and

Bernstein Center for Computational Neuroscience Munich

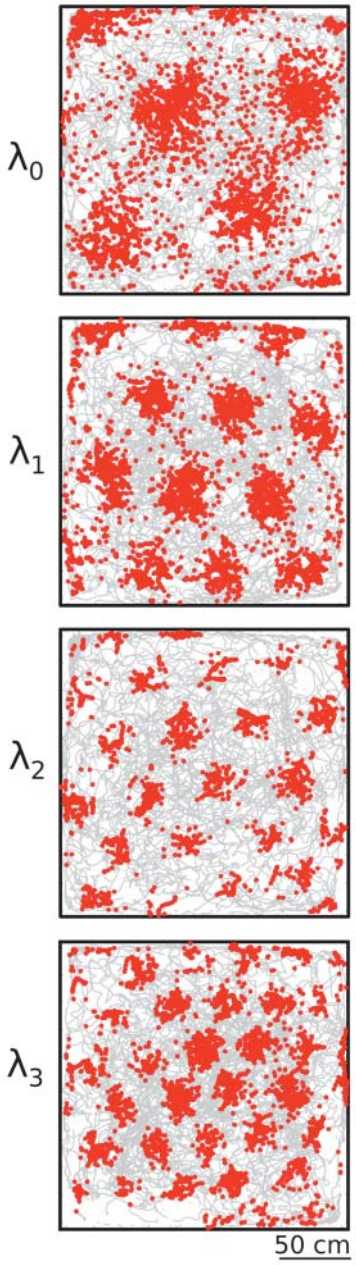
Grid-cell activity

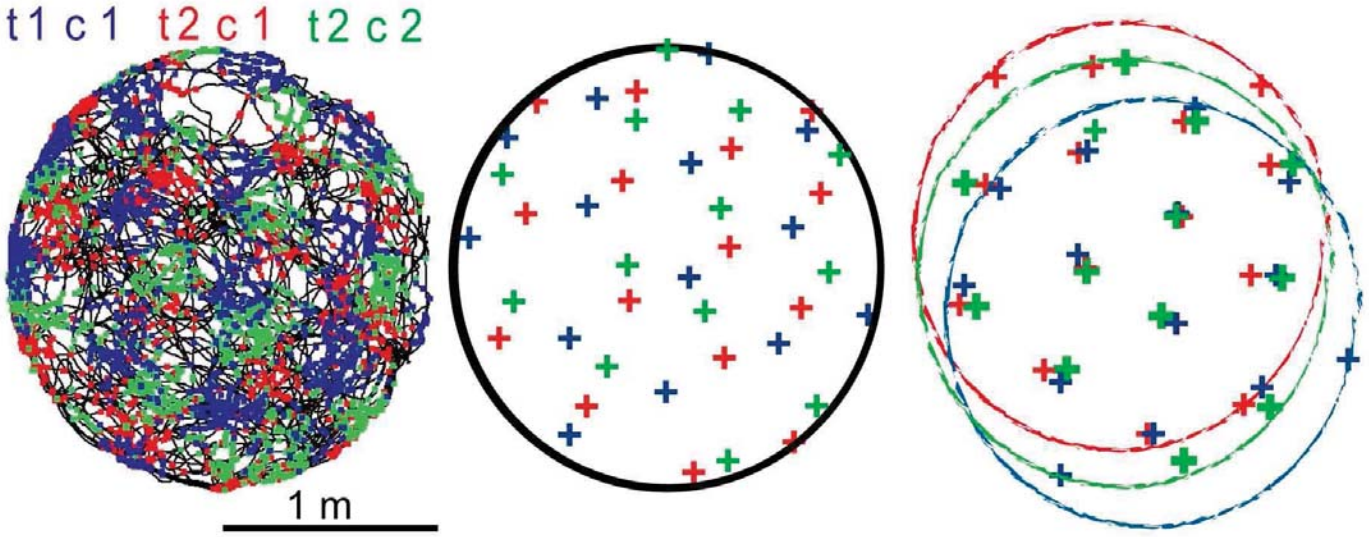


Hafting et al. (2005)

Grid cells in Medial Entorhinal Cortex (MEC)

Salient grid-cell features





Hafting et al. (2005)

Neighboring cells:
- similar spatial frequency
- similar orientation
- different spatial phase

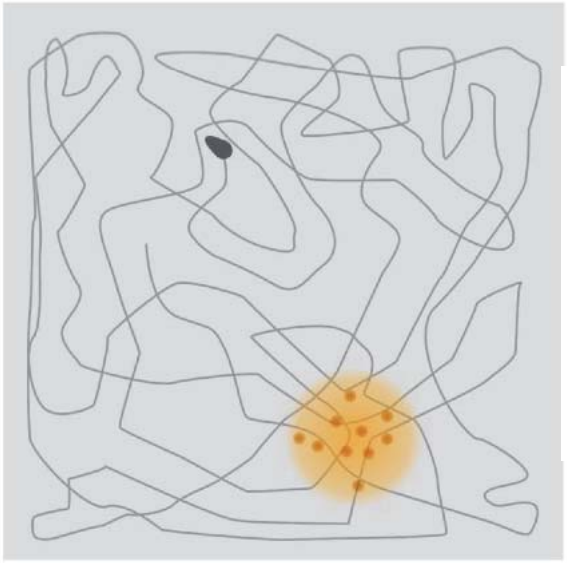
Grid scale increases in steps from dorsal to ventral

Stensola et al. (2012)

→ Grid cell „modules“ (4–10?)

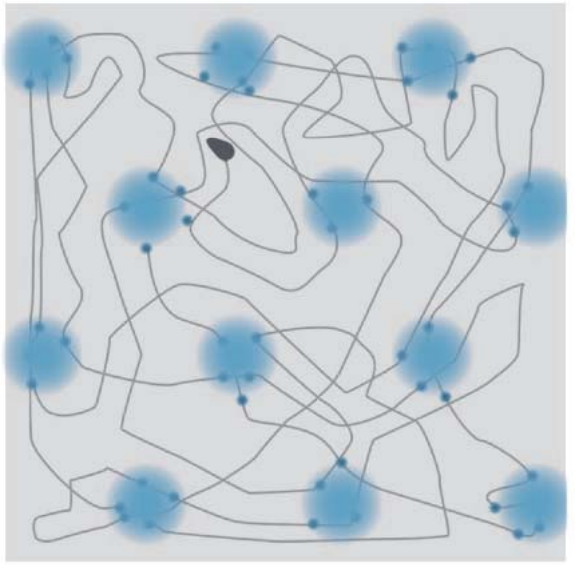
A metric for space

The Nobel Prize in Physiology or Medicine 2014



John O'Keefe

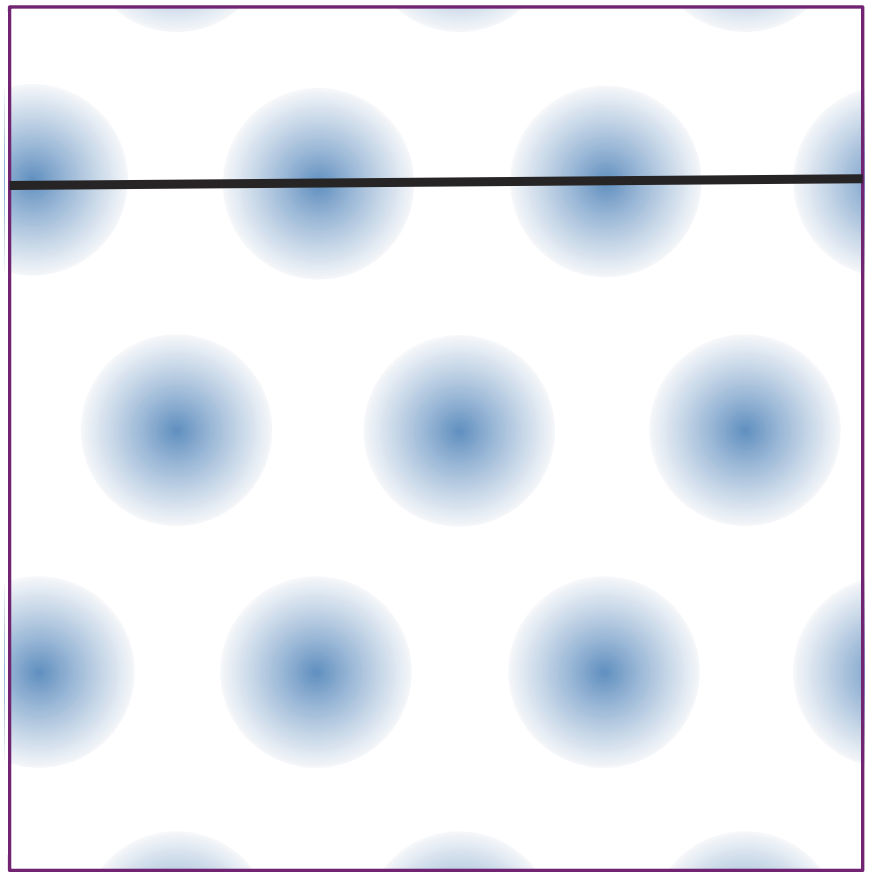
May-Britt Moser and
Edvard I. Moser



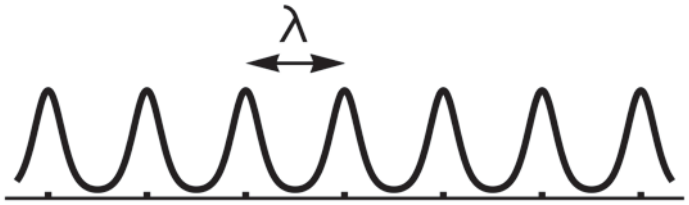
The Nobel Committee for Physiology or Medicine:

„The grid system provides a solution to measuring movement distances and adds a metric to the spatial maps in hippocampus ... an internal GPS“

A metric for space?



Periodic firing field Ω(x)



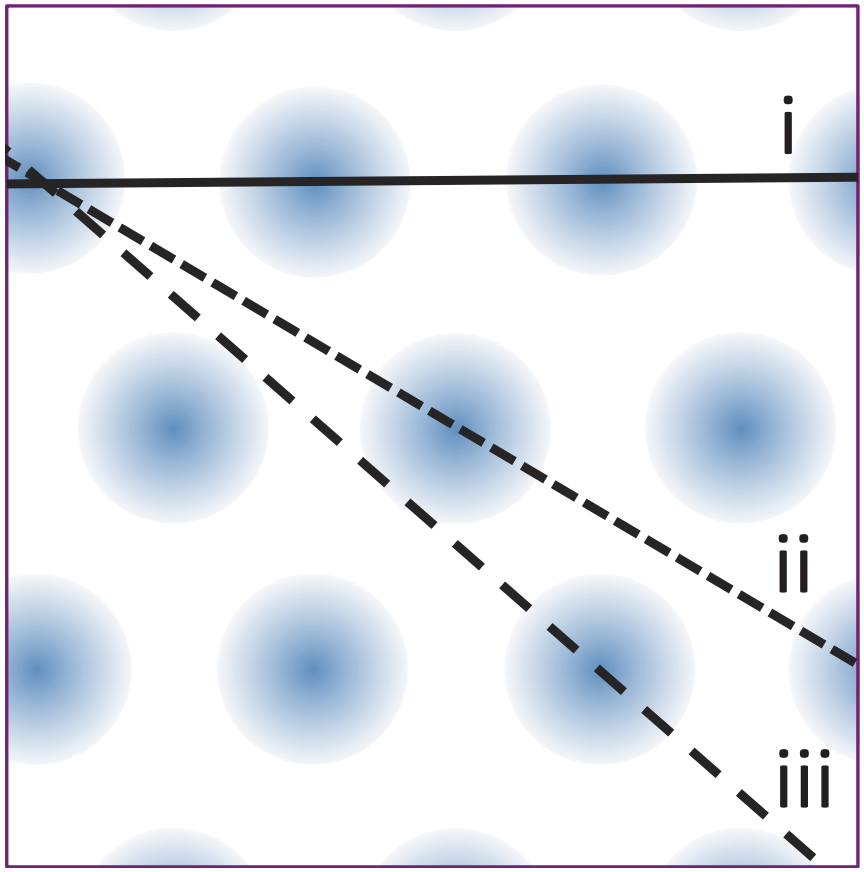
von Mises tuning:

$$\Omega_j(x) = \mathrm{n}\, e^{\kappa\,[\cos(2\pi/\lambda_j(x - c_j)) - 1]}$$

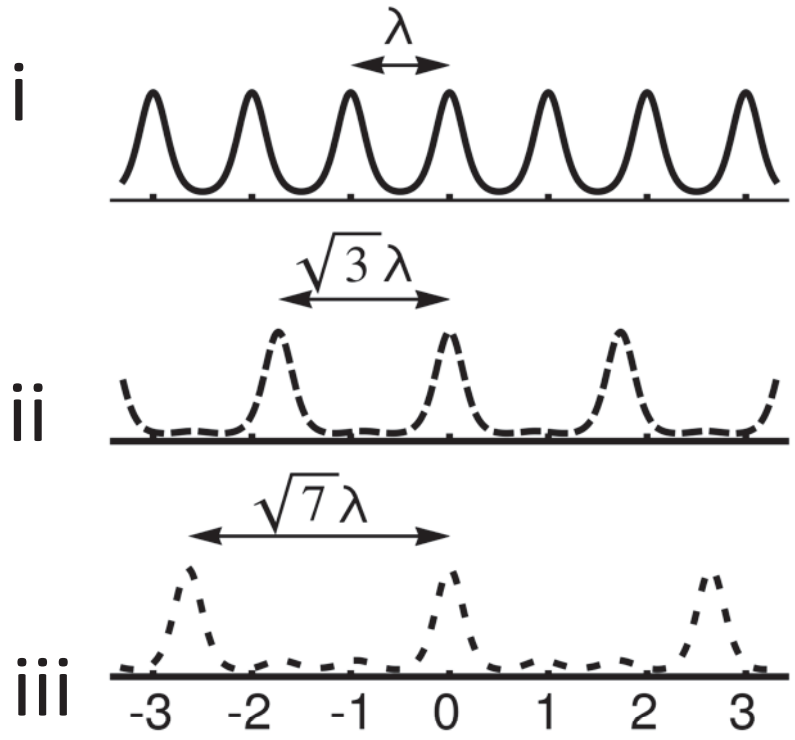
Four parameters

- grid scale λ
- relative field width (via κ)
- maximum spike count n in a certain time window (e.g. θ cycle)
- spatial phase c_j

A metric for space?



Periodic firing field Ω(x)



A single grid cell *cannot* represent metric relations.

What about local grid cell populations?

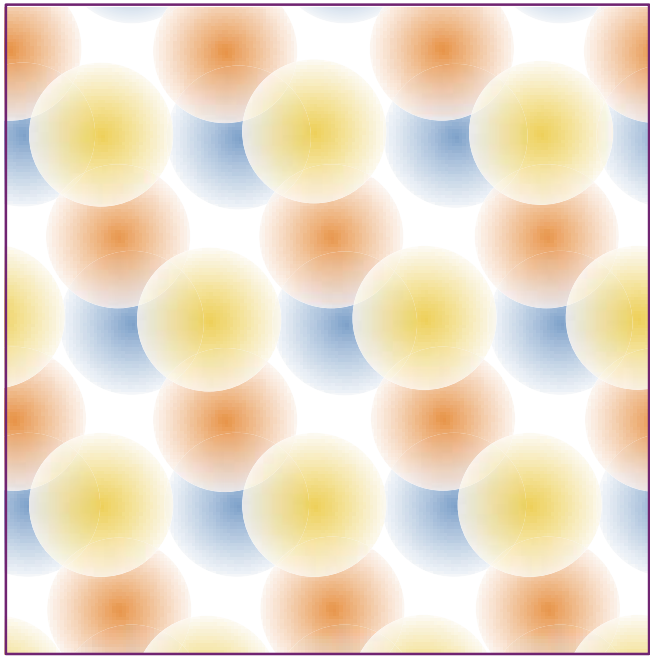
Encoding spatial location

Grid cells within one module:

(a) aligned orientations

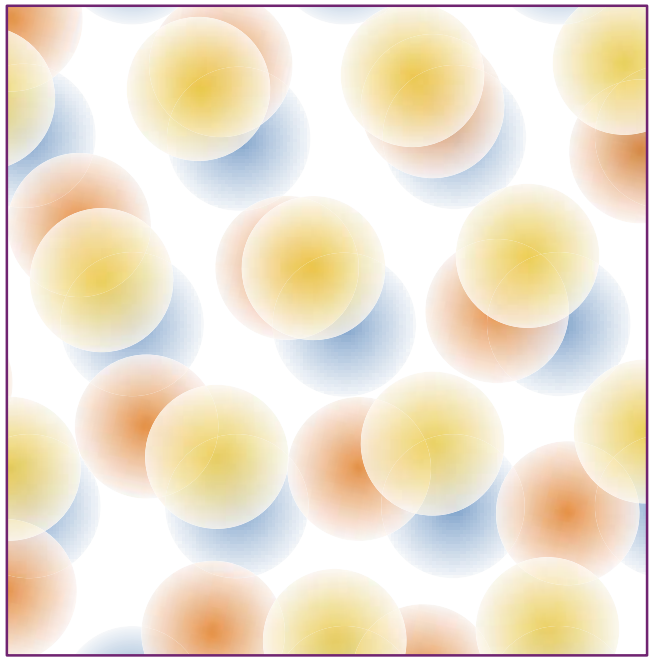
(b) similar grid scale

(c) Spatial phase shifts between grid cells



A single grid-cell module *cannot* represent spatial location.

Modules *cannot* operate independently.

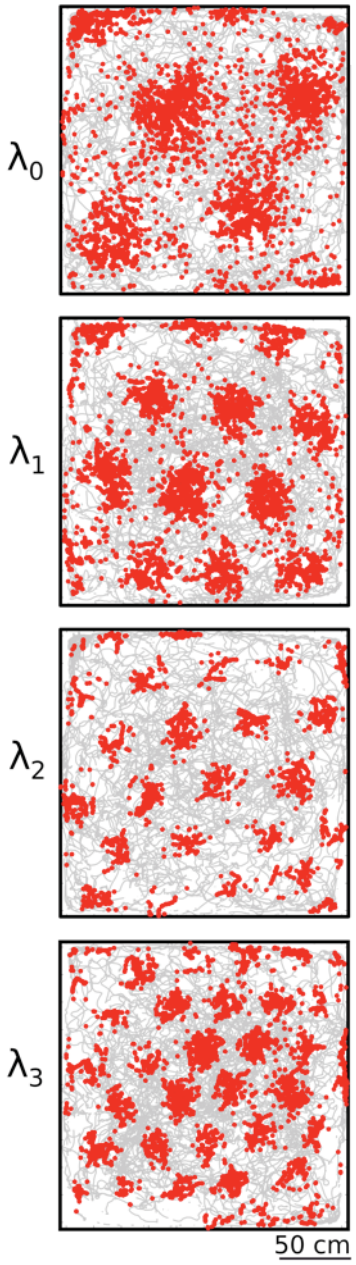


The puzzle: single modules could encode space if (a) or (b) did *not* hold.

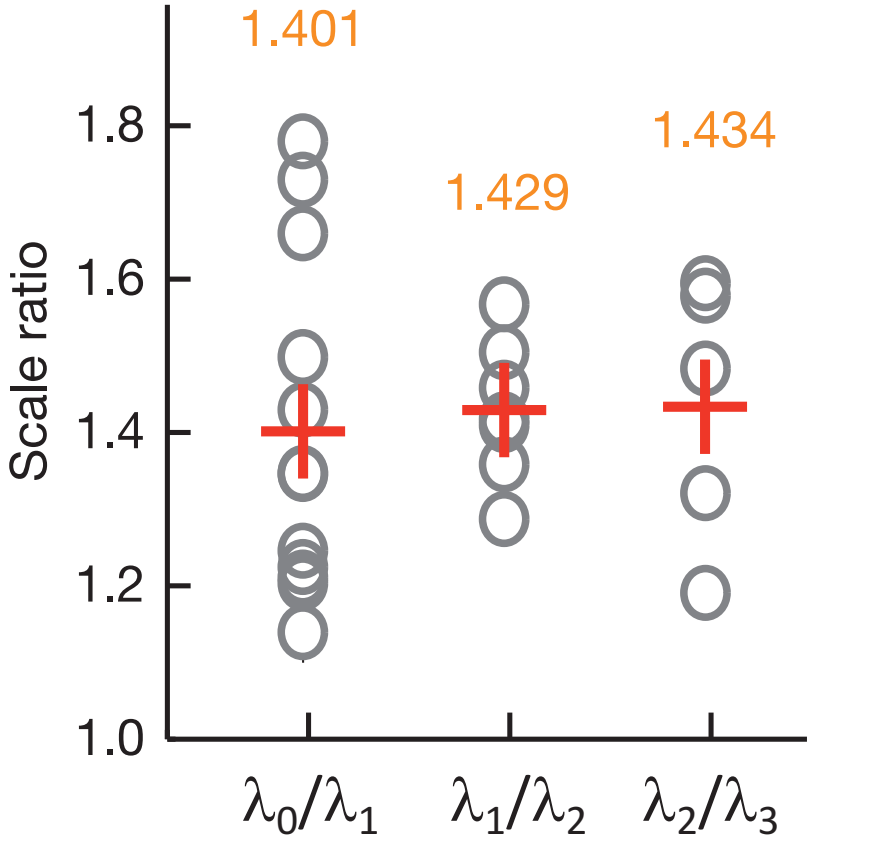
Decoding across grid scales – but how?

Advantage of aligned axes in a module?

Encoding spatial location: Scale ratio s



Scale ratio
$s = \lambda_i/\lambda_{i+1}$
is constant.



(Stensola et al. Nature 2012)

Numerical value: s ~ 1.42 (Barry et al. 2007: s ~ 1.7)

Why are these values around 1.5 ?

How to read the grid code?

Posterior (independent Poisson neurons):

$$P\left(\vec{x}|\boldsymbol{n}\right) \sim \prod_{j=1}^{N} \exp\left[n_j \ln\left(\Omega_j(\vec{x})\right)\right]$$

The most likely position: $\max P(x|\mathbf{n})$

$\rightarrow$ maximum likelihood decoding

- usual approach: numerical optimization
- biological implementation would require learning

Is there a biologically plausible alternative?

How to read the grid code?

von Mises tuning:

$$\Omega_j(x) = n_{max}\, e^{\kappa\,[\cos(2\pi/\lambda_j(x-c_j))-1]}$$

Posterior (independent Poisson neurons):

$$P\left(\vec{x}|\boldsymbol{n}\right) \sim \prod_{j=1}^{N} \exp\left[n_j \ln\left(\Omega_j(\vec{x})\right)\right]$$

is also von Mises

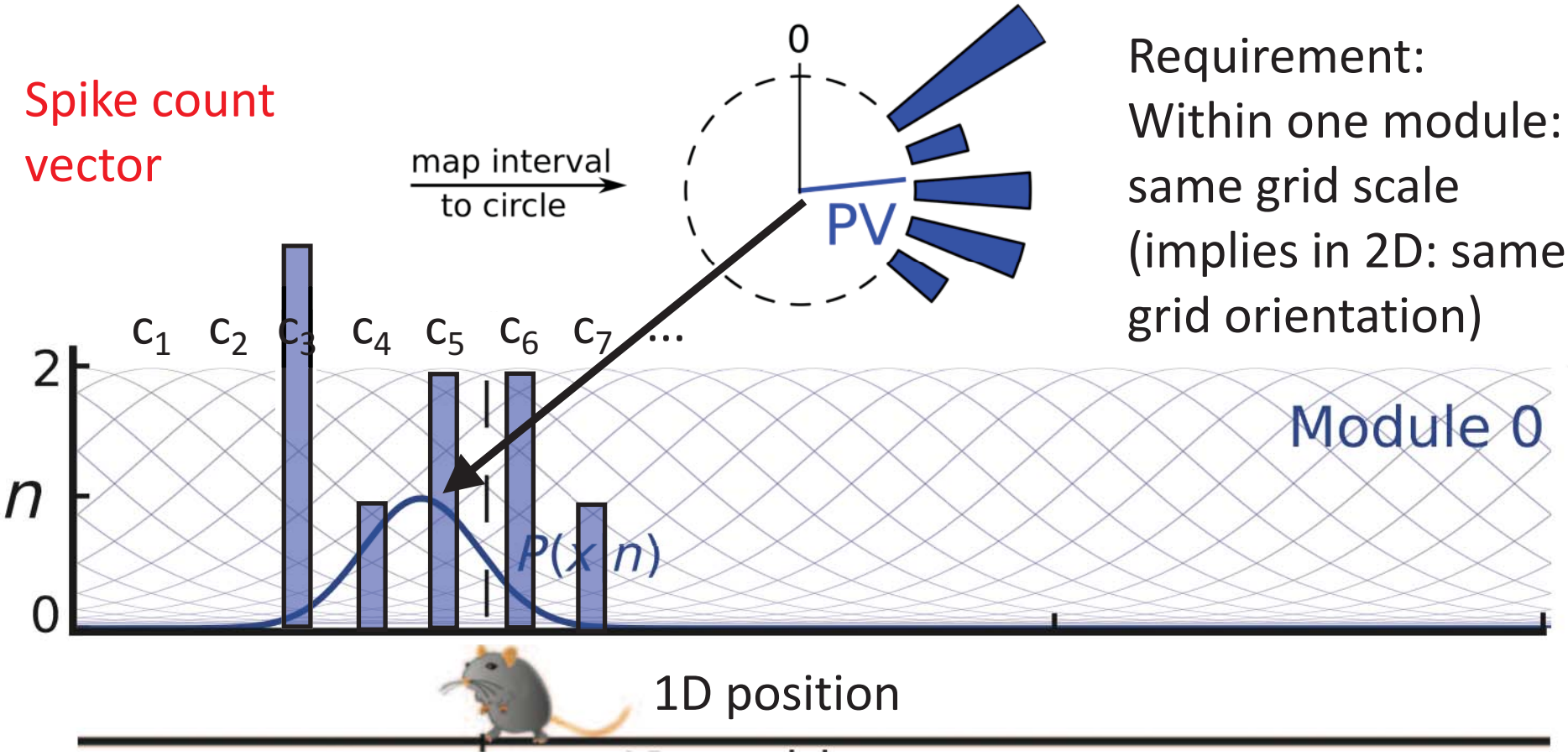
Maximum likelihood position estimate:

$$x_0 = \frac{\lambda_0}{2\pi} \arg\left(\sum_{j=1}^{M_0} n_j \exp(ic_j)\right)$$

Population vector

Requirement:
Within one module:
same grid scale
(implies in 2D: same grid orientation)

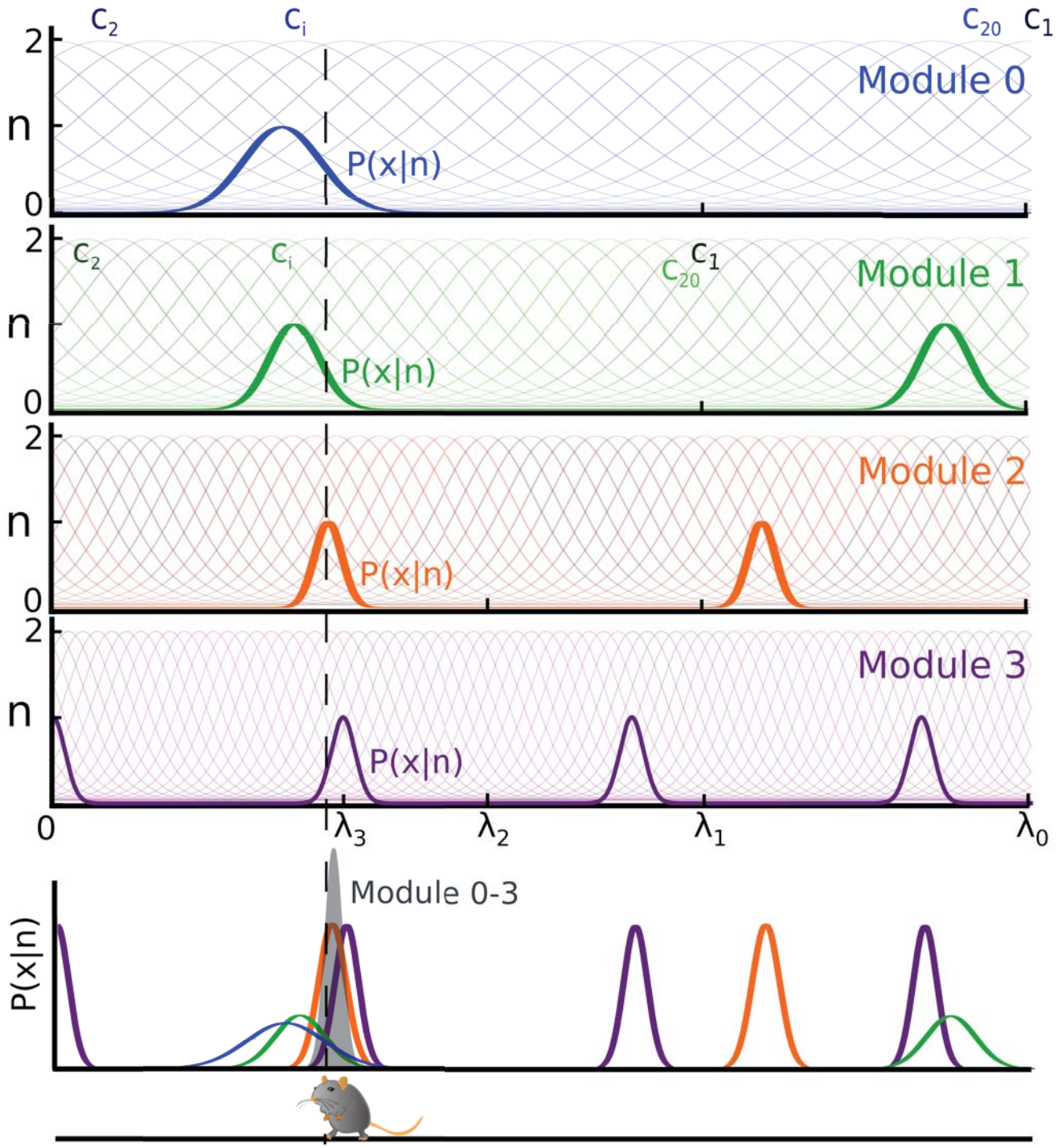
Spike count vector



Population vector decoding

across multiple spatial scales

Refinements at λ_1, λ_2

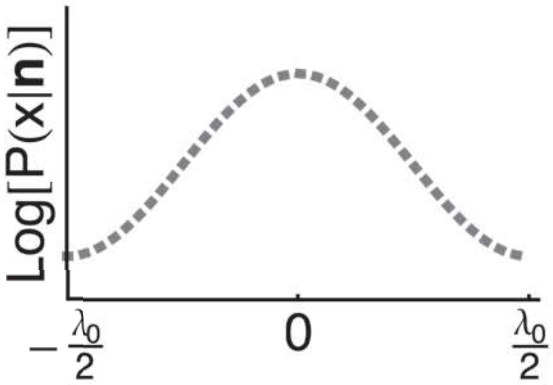


some predictions

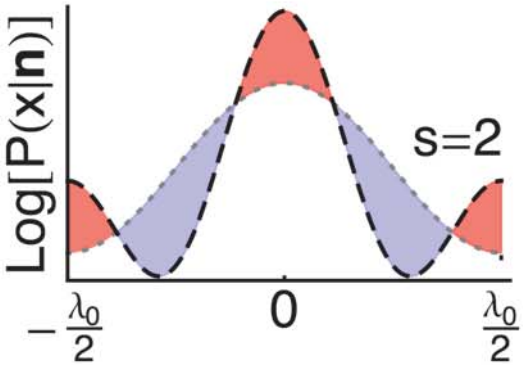
1D first,
generalizations later

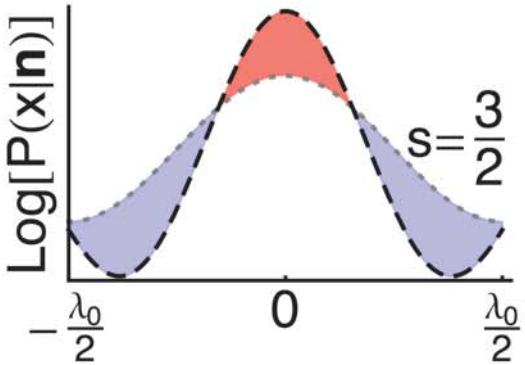
Scale ratio s

single module



two modules





"Catastrophic" errors
for s > 1.5

Predictions:

(a) s < 1.5

(b) silencing smallest module: mild decay of precision

(c) silencing intermediate modules: catastrophic errors

Goal-directed navigation

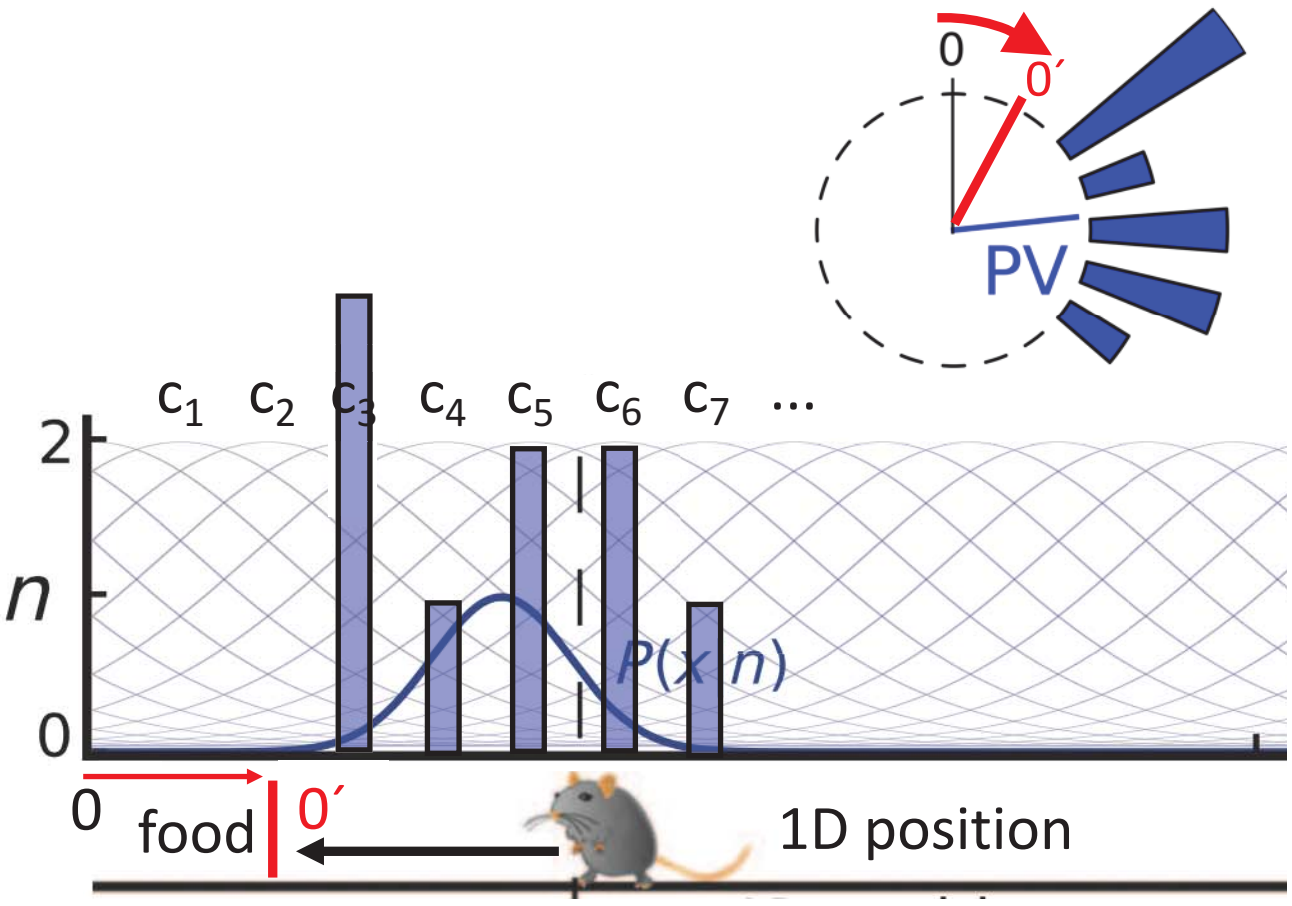
x_0: position relative to origin of allocentric coordinate system

Goal vector in egocentric coordinates?

Shift coordinate origin to goal location!

$$x_0 = \frac{\lambda_0}{2\pi} \arg\left(\sum_{j=1}^{M_0} n_j \exp(ic_j)\right)$$

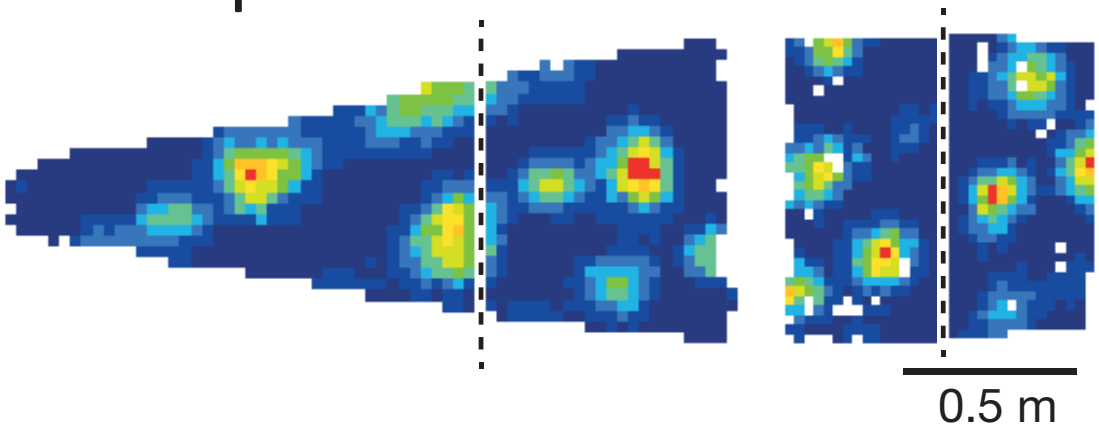
Multiply with sinusoidal gratings



Prediction: Downstream neurons (e.g. retrospinal cortex, posterior parietal cortex, prefrontal cortex):

Sinusoidal gain fields (as for gaze direction)

A metric for space?



Krupic et al.
Nature (2015)

(1) $d(\vec{x},\vec{y}) \geq 0$ with equality if and only if $\vec{x} = \vec{y}$

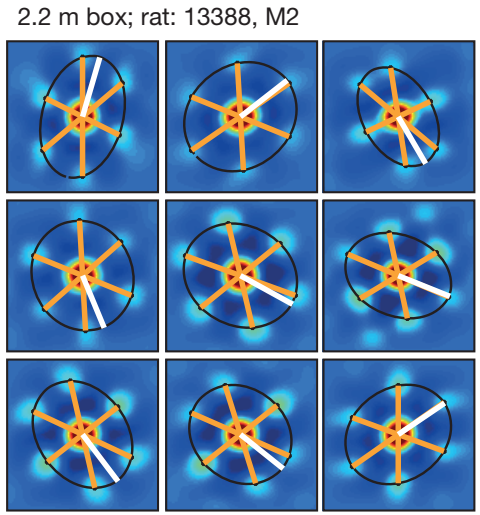
(2) $d(\vec{x},\vec{y}) = d(\vec{y},\vec{x})$ symmetry

(3) $d(\vec{x},\vec{y}) + d(\vec{z},\vec{y}) \geq d(\vec{x},\vec{z})$ triangle inequality

Ellipticity (even location dependent):

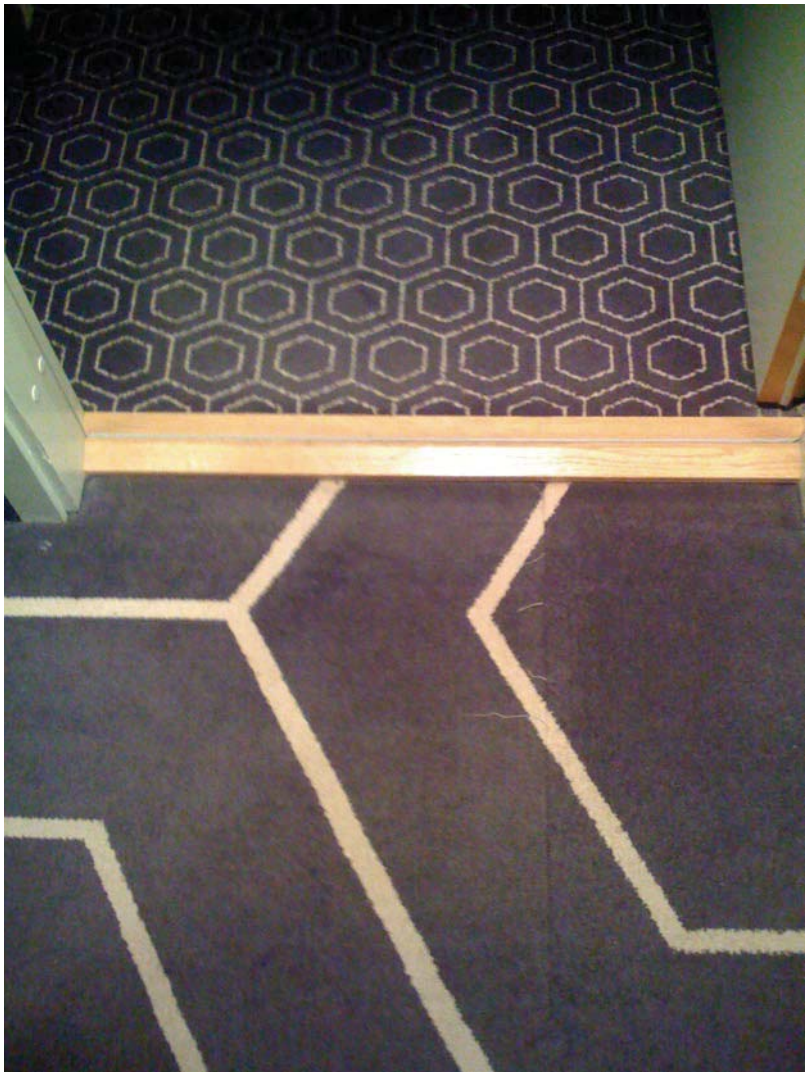
adjust readout

Note:
Distortions are homogeneous across population



Stensola et al.
Nature (2015)

Scandic Bakklandet, Trondheim

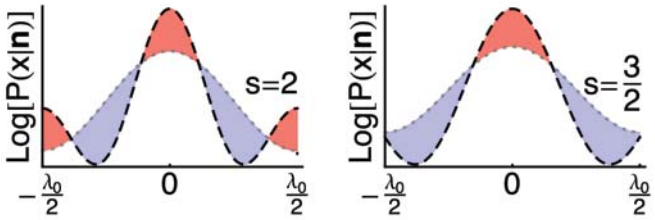


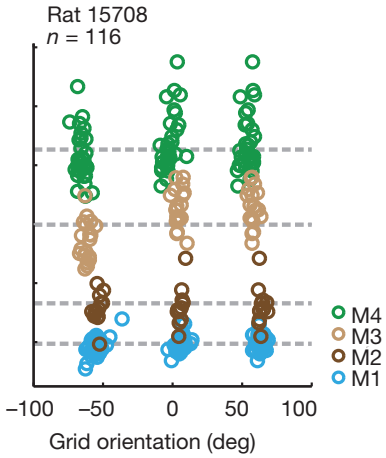
Summary of predictions

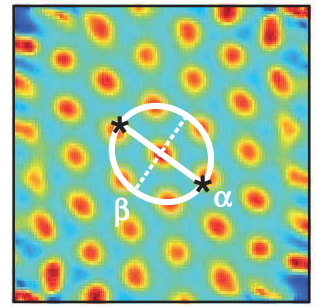
Stemmler et al. Sci Adv (2015)
advances.sciencemag.org/content/1/11/e1500816

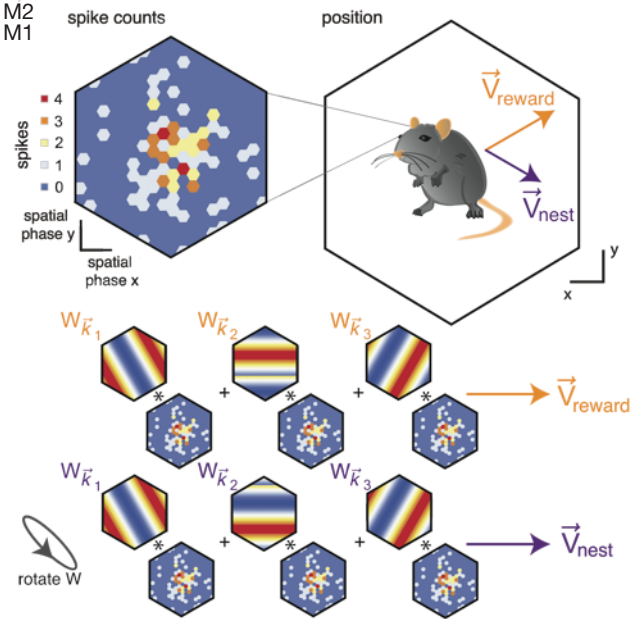
- fixed scale ratio s < 1.5
- silencing finest module: mild decay of precision
- silencing intermediate modules: “catastrophic“ errors
- tendency to align grid axes within and between modules
- downstream (retrospinal, post. parietal, prefrontal cortex ...):

  sinusoidal gain fields







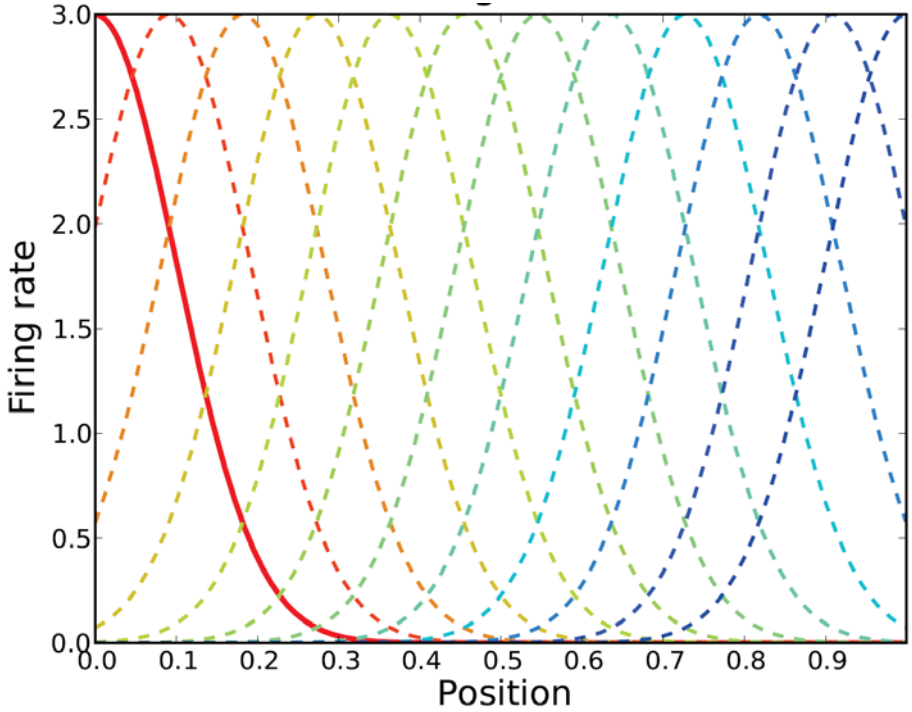


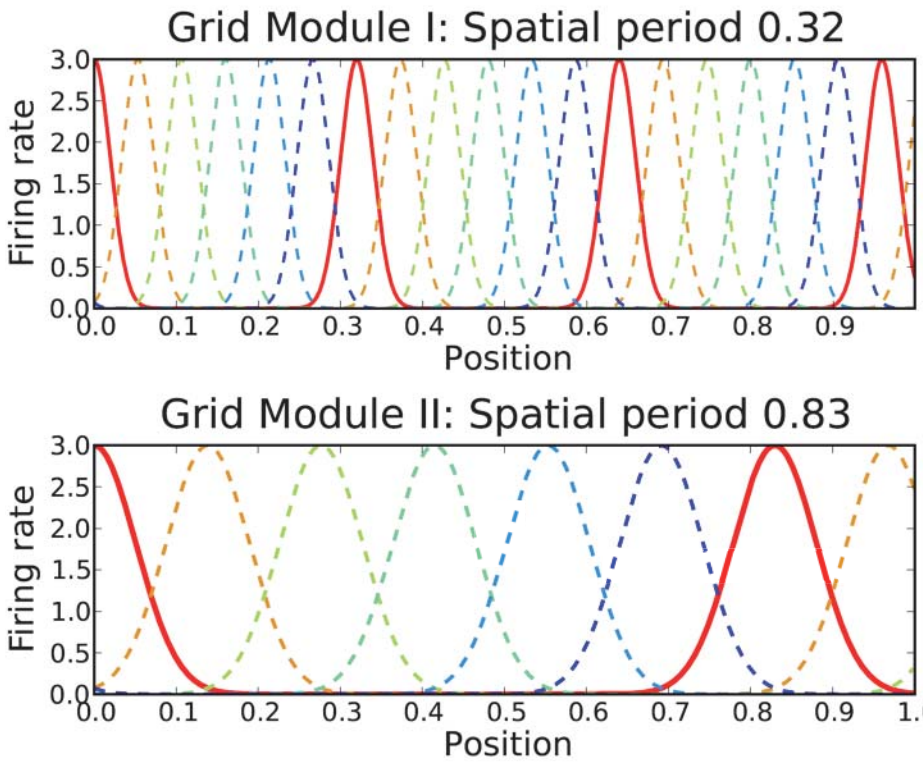
- (locally) distorted fields: adjust read out
- weaker synapses from fine modules
- beyond λ_0: integer arithmetic with error correction

Robots: Simulations: Vegard Edvardsen (CS, NTNU)

Encoding spatial location: Place vs. grid code

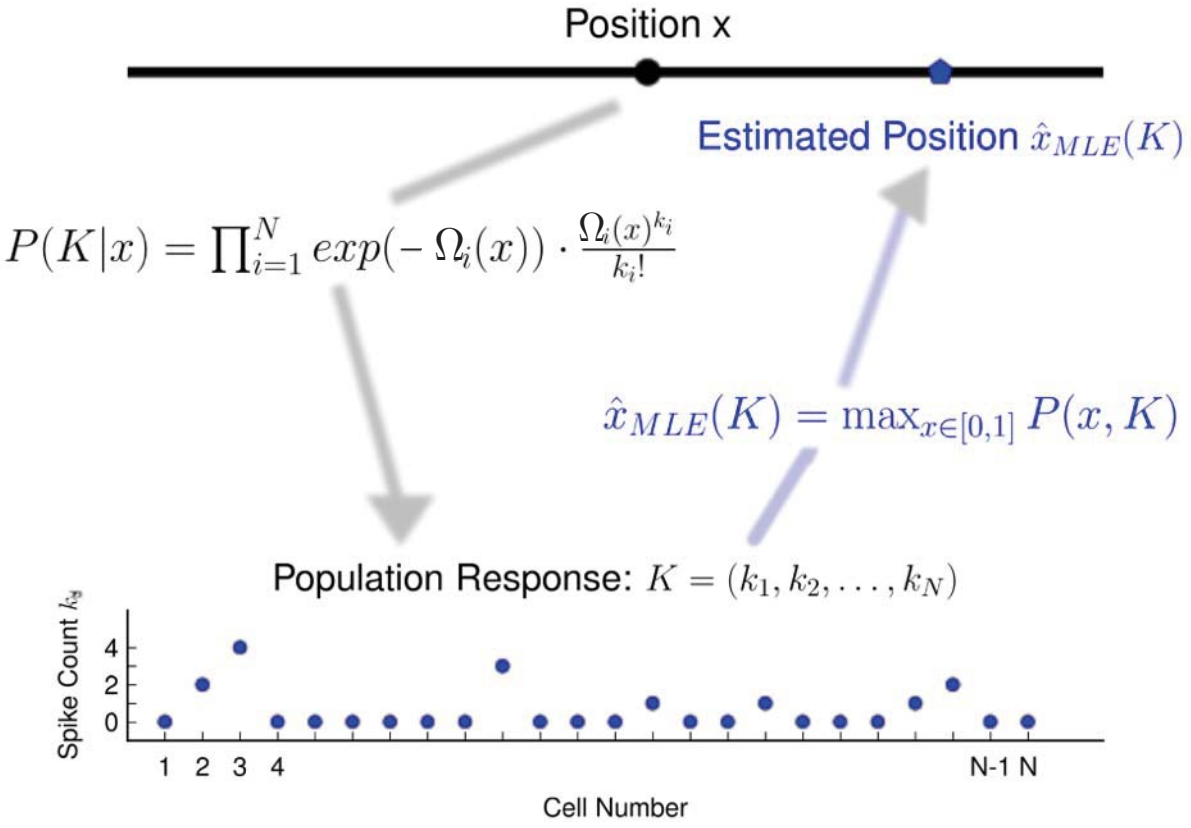
Tuning curves $\Omega_i(x)$





For the same total number of neurons,
which code is better suited to resolve spatial location?

Encoding spatial location: Place vs. grid code



Place code

Error ~ $[N\ f_{max}T]^{-1}$

Nested grid code
($\lambda_{i+1} = s\ \lambda_i$)

Error ~ $[M\ f_{max}T]^{-N/M}$

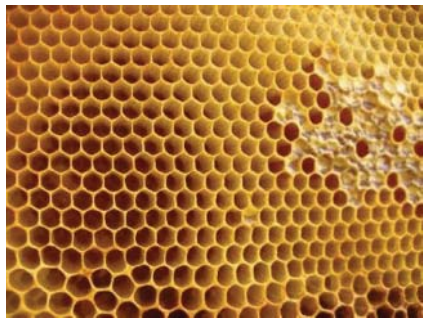
Grid codes outperform place codes.

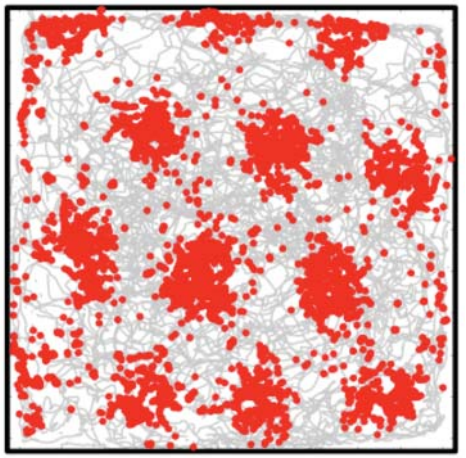
Prediction: Fixed scale ratio s (but no specific value for s)
[not predicted by / not advantageous for modular arithmetic]

Mathis et al. Neural Comp 2012, PRL 2012

Related: Wei et al. eLife 2015, Towse et al. Phil Trans R Soc B 2014

Optimal grids in 2D and 3D

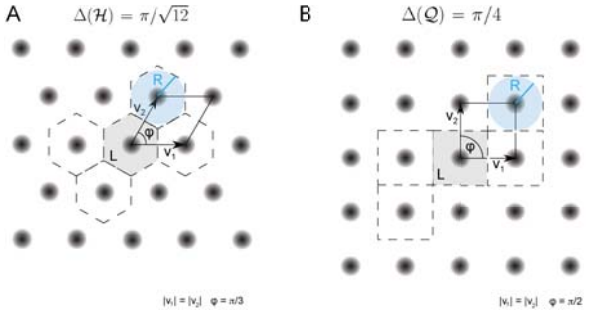




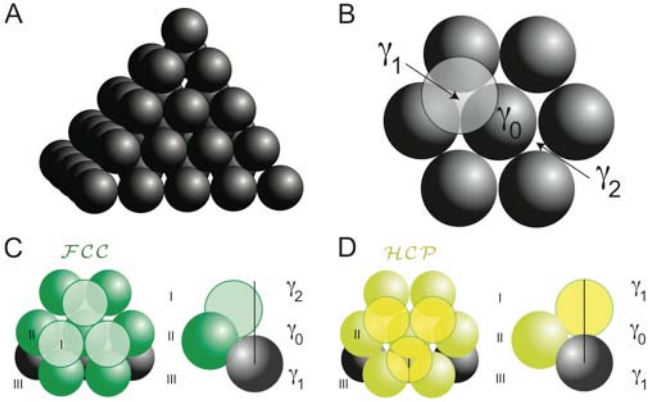
Closest packing in 3D: 1611 Kepler / 1831 Gauss / 1998 Hales

Grid cells: firing fields overlap cell-to-cell – not a trivial problem

Separated spherical fields … : Fisher information ≈ packing density



Prediction:
FCC/HCP in species
freely moving in 3D



Mathis et al.
eLife 2015

Yartsev & Ulanovsky
Isotropic place fields